4317. Exam 1

Friday, October 5, 2018 1:33 PM

Last name:

First name:

BU or BG:

Which 5 problems do you want graded:

Problem 1 (5 points): Let $E = \{x \in \mathbb{R}: x \geq 1\}$. For all $x, y \in E$, we define $d(x,y) = \left|\frac{1}{x} - \frac{1}{y}\right|$.

a) Prove that d is a distance in E. (1 points).

In the rest of the questions in this problem, we are using this distance.

b) Is the sequence $x_n = n$ bounded? (Prove your answer). (1 points)

c) Is the sequence $x_n = n$ Cauchy? (Prove your answer). (1 points)

d) Does the sequence $x_n = n$ converge? (Prove your answer). (1 points)

a) Is this space complete? (Prove your answer). (1 points)

Problem 2 (5 points): Show that the subset of $\mathbb{R}^2$ by $\{(x,y) \in \mathbb{R}^2: x^2 \geq \ y^2 \ \ \mathbb{R}\}$ is closed.

Problem 3 (5 points): Let x_n be a sequence in a metric space E. Assume that the following are satisfied:

a) For all positive integers N, the set $\{x_n : n \geq N\}$ is closed.

b) The sequence x_n is Cauchy.

c) E is compact.

Prove that there exists $x \in E$ and a subsequence x_{n_i} such that $x_{n_i} = x$ for all i.

Problem 4 (5 points): Let d and d' be two distances on the same set E so that the following is satisfied: If $U \subset E$ is open with the distance d, then it is also open with the distance d'.

Let $x \in E$. Prove that for all $\varepsilon > 0$, there exits $\delta > 0$ such that $d'(x,y) < \delta$ implies $d(x,y) < \varepsilon$.

Problem 5 (5 points): Let $S \subset \mathbb{R}$. Let $x_n \in S$. Assume x_n converges to x but x is not in S. Assume that S is connected and x_n has an increasing subsequence (i.e. $x_{n_{i+1}} \geq x_{n_i}$). Prove that for all n, there exists $m > n$ such that $x_m > x_n$.

Problem 6 (5 points): Let $S = \left\{\frac{1}{n} : n \text{ in an positive integer}\right\} \cup \{0\}$. Prove that S is compact.